Nobel lecture
Angrist and Imbens' methodological contributions

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Motivation: heterogeneity and causality

- Main idea: people react differently to "stimuli", treatments etc.
- Heterogeneous reactions to "treatments": vaccinations, taxes, education etc.
- Also: heterogeneous reactions to the instrument.
- Angrist and Evans (1998, AE98 below): some parents want a third child if the first two are of the same sex, some parents don't.
- Question: how does such heterogeneity affect the interpretation of usual estimators?

- Binary treatment D. Examples:
  - Vaccination ($D = 1$) or not ($D = 0$);
  - AE98: having three or more children ($D = 1$) vs only two ($D = 0$)
- Binary instrument Z. Examples:
  - Experiments with imperfect compliance: $Z = 1$ (resp. $Z = 0$) if allocated to the treatment group (resp. control group).
  - In AE98: $Z = 1$ if the first two children have the same sex, $Z = 0$ otherwise.
- Outcome variable Y. In AE98: hours worked, work or not.
- The instrumental variable estimand is equal to

$$\beta_Z = \frac{E(Y|Z=1) - E(Y|Z=0)}{E(D|Z=1) - E(D|Z=0)}.$$

- IA94's question: how can we relate β_Z to causal effect(s) of D on Y?

- To properly define causality, we introduce potential variables, following Neyman (1923) and Rubin (1974).
- Two potential treatments: $D(0)$ = treatment if $Z = 0$, $D(1)$ =treatment if $Z = 1$.
- For each individual, we only observe $D := D(Z)$, namely $D(0)$ if $Z = 0$ and $D(1)$ if $Z = 1$.
- Potential outcomes: $Y(0)$ =outcome absent the treatment, $Y(1)$ = outcome with the treatment.
- For each individual, we only observe $Y := Y(D)$, namely $Y(0)$ if $D = 0$ and $Y(1)$ if $D = 1$.
- Causal effect of D on Y: $Y(1) - Y(0)$. It may vary from one indiv. to another.

Assumptions in IA94: independence and monotonicity

- Independence of the instrument:

$$Z \perp\!\!\!\perp (D(0), D(1), Y(0), Y(1)), \tag{1}$$

  where "$\perp\!\!\!\perp$" means "independent of", in the probabilistic sense.

- Independence is credible:
  - in randomized experiments, since Z is drawn independently of indiv's characteristics;
  - in natural experiments (e.g., in AE98), where "nature draws Z".
- Monotonicity:

$$D(1) \geq D(0) \quad \text{almost surely.} \tag{2}$$

- Monotonicity:
  - is credible In randomized experiments if individuals in the control group cannot be treated ($D(0) = 0$);
  - may not hold in randomized experiments with encouragement designs;
  - holds in AE98 if no parents strictly want two boys or two girls.

Main result of IA94

Theorem

Under A1-A2 and if $E(D|Z=1) > E(D|Z=0)$, *we have*

$$\beta_Z = E[Y(1) - Y(0)|D(1) > D(0)].$$

- $E[Y(1) - Y(0)|D(1) > D(0)]$ called "local average treatment effect" (LATE).
- "Local" because we identify the average effect of the treatment for a subpopulation only, that of "compliers" $(D(1) > D(0))$.
- Changing the instrument changes the population of compliers, and thus the value of β_Z in general.

- If independence fails, we do not identify a causal effect anymore in general.
- In AE98: could fail b/c Z affects the budget set of households (Rosenzweig and Wolpin, 2000).
- If monotonicity fails, let $C = \{D(1) > D(0)\}$, $F = \{D(1) < D(0)\}$ ($F =$"defiers"). Then:

$$\beta_Z = \lambda E[Y(1) - Y(0)|C] + (1 - \lambda)E[Y(1) - Y(0)|F],$$

with $\lambda = P(C)/[P(C) - P(F)] > 1$.

$\Rightarrow$ β_Z could be < 0 even if $Y(1) > Y(0)$ for everyone.

- Yet, de Chaisemartin (2017) shows that β_Z still identifies a causal effect under a much weaker "Compliers - Defiers" condition.

- Treatment is often non-binary: number of children, education, level of taxes etc.
- Result extended to ordered and continuous (still scalar) D by Angrist and Imbens (1995) and Angrist, Graddy and Imbens (2000).
- With a continuous D:

$$\beta_Z = E\left[W \times \frac{\partial Y}{\partial d}(D)\right],$$

  where W is a random weight: $W \geq 0$ and $E(W) = 1$.
- Discrete, unordered treatment: see Heckman and Pinto (2018) and Lee and Salanié (2018).

- Extension to a non-binary Z: Heckman and Vytlacil (2007).
- Assume $D(z) = 1\{P(z) \geq U\}$ for some $P(.)$, $U \sim \mathcal{U}([0,1])$ and $(U, Y(0), Y(1)) \perp\!\!\!\perp Z$.
- Then:

$$\frac{\partial E(Y|P = p)}{\partial p}\Big|_{p_0} = E[Y(1) - Y(0)|U = p_0],$$

  where $P := P(Z)$. $E[Y(1) - Y(0)|U = p_0]$ = "marginal treatment effect".
- The LATE can be expressed as a function of MTE.
- $\partial E(Y|P = p)/\partial p$ called "local instrumental variable estimator".

Still other extensions of IA94

- "Fuzzy" regression discontinuity designs (Hahn et al, 2001).
- Assume that above or below a threshold, people more likely to be treated.
- Example: grade retention if gpa below 10/20.
- Assume (basically):
  - Z continuous;
  - $z \mapsto E[D|Z = z]$ discontinuous at z_0;
  - $z \mapsto E[Y(d)|Z = z]$.

  Then:

$$\frac{E(Y|Z = z_0^+) - E(Y|Z = z_0^-)}{E(D|Z = z_0^+) - E(D|Z = z_0^-)} = E[Y(1) - Y(0)|Z = z_0, D(z_0^+) > D(z_0^-)]$$

- "Fuzzy" difference-in-differences (de Chaisemartin and DH, 2018).

Concluding remarks

- Profound influence of IA94 on the methodological study of causality.
- Many other important works of G. Imbens (some with J. Angrist) on treatment effects:
  - Study of matching estimators;
  - regression discontinuity designs;
  - (nonlinear) difference-in-differences;
  - quantile instrumental variable methods;
  - Recently, with S. Athey: use of machine learning tools for causality.